1 George throws a ball at a target 15 times.
Each time George throws the ball, the probability of the ball hitting the target is 0.48

The random variable X represents the number of times George hits the target in 15 throws.

(a) Find

(i) $\mathrm{P}(X = 3)$

(ii) $\mathrm{P}(X \geqslant 5)$

(3)

George now throws the ball at the target 250 times.

(b) Use a normal approximation to calculate the probability that he will hit the target more than 110 times.

(3)

a) $X \sim B(15, 0.48)$ 1

(i) $P(X=3) = 0.019668...$
$= 0.0197$ (3sf) 1

(ii) $P(X \geq 5) = 1 - P(X \leq 4)$
$= 0.92013...$
$= 0.920$ (3sf) 1

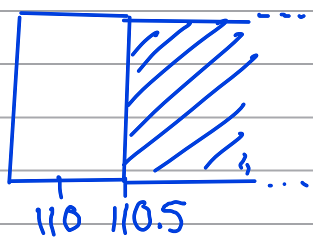
b) let Y be the number of times George hits the target.

$\mu = np = 250 \times 0.48 = 120$
$\sigma = \sqrt{np(1-p)} = \sqrt{62.4}$

$Y \sim N(120, \sqrt{62.4}^2)$ 1

$P(X > 110)$

$\approx P(Y > 110.5)$ 1



$= 0.88544...$
$= 0.885$ (3sf) 1

2 A manufacturer uses a machine to make metal rods.

The length of a metal rod, L cm, is normally distributed with

- a mean of 8 cm
- a standard deviation of x cm

Given that the proportion of metal rods less than 7.902 cm in length is 2.5%

(a) show that $x = 0.05$ to 2 decimal places.

(2)

(b) Calculate the proportion of metal rods that are between 7.94 cm and 8.09 cm in length.

(1)

The **cost** of producing a single metal rod is 20p

A metal rod

- where $L < 7.94$ is **sold** for scrap for 5p
- where $7.94 \leqslant L \leqslant 8.09$ is **sold** for 50p
- where $L > 8.09$ is shortened for an extra **cost** of 10p and then **sold** for 50p

(c) Calculate the expected profit per 500 of the metal rods.
Give your answer to the nearest pound.

(5)

The same manufacturer makes metal hinges in large batches.

The hinges each have a probability of 0.015 of having a fault.

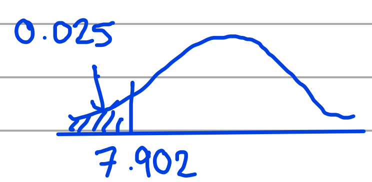
A random sample of 200 hinges is taken from each batch and the batch is accepted if fewer than 6 hinges are faulty.

The manufacturer's aim is for 95% of batches to be accepted.

(d) Explain whether the manufacturer is likely to achieve its aim.

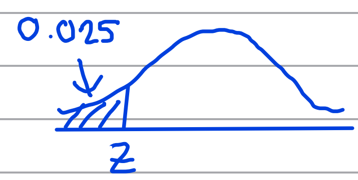
(4)

$L \sim N(8, x^2)$



$P(L < 7.902) = 0.025$

$P\left(Z < \frac{7.902 - 8}{x}\right) = 0.025$



$\therefore \frac{7.902 - 8}{x} = -1.96$ ①

$\Phi^{-1}(0.025) = -1.96$

$\frac{7.902 - 8}{x} = -1.96$

$x = \frac{7.902 - 8}{-1.96} = 0.05 \text{ (2dp)}$ (1)

b) $P(7.94 < L < 8.09) = P(L < 8.09) - P(L < 7.94)$
$= 0.8490\ldots$
$= 0.849 \text{ (3dp)}$ (1)

c) cost of producing 500 rods $= 500 \times 0.2 = £100$

Number sold for scrap:
$P(L < 7.94) = 0.115 \text{ (3sf)}$ (1)
$0.115 \times 500 = 57.5$ rods sold for scrap

Each sold for 5p: $57.5 \times 0.05 = £2.88$

Number sold for normal price:
$P(7.94 < L < 8.09) = 0.849$
$0.849 \times 500 = 424.5$ rods sold for normal price

Each sold for 50p: $424.5 \times 0.5 = £212.25$

Number shortened:
$P(L > 8.09) = 0.0359$ (1)
$0.0359 \times 500 = 17.97$ rods shortened (1)

Each make profit of 50p - 10p = 40p: $17.97 \times 0.4 = £7.19$

Total profit $= £2.88 + £212.25 + £7.19 - £100$ (1)
$= £122.32$ (1)

d) let X be the number of hinges that are faulty

$X \sim B(200, 0.015)$ 1

$P(X<6) = P(X \leq 5) = 0.9176... < 0.95$ 1 1

So it is expected that 91.8% of batches will be accepted. Therefore the manufacturer is unlikely to achieve their aim as $91.8 < 95$ 1

3. A machine fills packets with sweets and $\frac{1}{7}$ of the packets also contain a prize.
The packets of sweets are placed in boxes before being delivered to shops.
There are 40 packets of sweets in each box.

The random variable T represents the number of packets of sweets that contain a prize in each box.

(a) State a condition needed for T to be modelled by $B(40, \frac{1}{7})$

(1)

A box is selected at random.

(b) Using $T \sim B(40, \frac{1}{7})$ find

(i) the probability that the box has exactly 6 packets containing a prize,

(ii) the probability that the box has fewer than 3 packets containing a prize.

(2)

Kamil's sweet shop buys 5 boxes of these sweets.

(c) Find the probability that exactly 2 of these 5 boxes have fewer than 3 packets containing a prize.

(2)

Kamil claims that the proportion of packets containing a prize is less than $\frac{1}{7}$

A random sample of 110 packets is taken and 9 packets contain a prize.

(d) Use a suitable test to assess Kamil's claim.
You should

- state your hypotheses clearly
- use a 5% level of significance

(4)

a) The probability of a packet containing a prize is constant. 1

b) $T \sim B(40, \frac{1}{7})$

(i) $P(T=6) = 0.1727\ldots = 0.173$ (3 s.f.) 1

(ii) $P(T<3) = P(T \leqslant 2)$

$= 0.061587\ldots = 0.0616$ (3 s.f.) 1

(c) Let r.v. K = number of boxes with fewer than 3 packets containing a prize.

$K \sim B(5, 0.0615...)$ 1

$\therefore P(K=2) = 0.031344... = 0.0313$ (3 s.f.) 1

d) Let r.v. X = number of packets containing a prize.

$X \sim B = (110, \frac{1}{7})$ 1

$H_0 : p = \frac{1}{7}$, $H_1 = p < \frac{1}{7}$ 1

$P(X \leq 9) = 0.038292...$ (which is < 0.05) 1

$\therefore$ reject H_0 since there is evidence to support Kamil's claim. 1

4. Xian rolls a fair die 10 times.

The random variable X represents the number of times the die lands on a six.

(a) Using a suitable distribution for X, find

(i) $P(X = 3)$

(ii) $P(X < 3)$

(3)

a die has 6 sides, so
P(lands on six) = 1/6

Xian repeats this experiment each day for 60 days and records the number of days when $X = 3$

(b) Find the probability that there were at least 12 days when $X = 3$

(3)

(c) Find an estimate for the total number of sides that Xian will roll during these 60 days.

(1)

(d) Use a normal approximation to estimate the probability that Xian rolls a total of more than 95 sixes during these 60 days.

(4)

a) $X \sim B(10, 1/6)$ 1

(i) $P(X=3) = 0.155045...$
$= 0.155$ (3sf) 1

(ii) $P(X<3) = P(X \leq 2) = 0.775226...$
$= 0.775$ (3sf) 1

b) Let D = the number of days when X = 3

$D \sim B(60, 0.155...)$ 1

$P(D \geq 12) = 1 - P(D \leq 11)$ 1
$= 1 - 0.78819...$
$= 0.212$ (3sf) 1

c) Xian rolls 10 dice 60 times = 600 dice rolled

sixes 1/6 of the time = $600 \times \frac{1}{6}$ = 100 sixes 1

d) let S be the normal approximation for D.

$\mu = np = 600 \times \frac{1}{6} = 100$

$\sigma = \sqrt{np(1-p)} = \sqrt{\frac{5}{6} \times 100} = \sqrt{\frac{250}{3}}$

$S \sim N\left(100, \sqrt{\frac{250}{3}}^{\,2}\right)$ (1) (1)

$P(D > 95) \approx P(S > 95.5) = 0.688976...$ (1)

$= 0.689$ (3sf) (1)